PDE control Dallas-style: Oil drilling & production

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Spong Fest
Length of a well: 100m–5km (300–15000 ft.)
Drill bit
Common instabilities

- Whirling oscillations (beam bending)
- Vertical oscillations
  - wave-induced mud pressure vibrations
  - and “bit bounce” (w/ 3-cone bit)
- Stick-slip oscillations (torsional)

\[
T(u_t(0,t))
\]

\[
U(t)
\]
Experimental data (stick-slip instability)
Model of angular displacement dynamics $u(x, t)$

**Torsional wave equation + B.C.**

\[ u_{tt}(x, t) = u_{xx}(x, t) \]

\[ u_x(1, t) = U(t) \quad \text{(torque input)} \]

\[ u_{tt}(0, t) = a u_x(0, t) + a T(u_t(0, t)) \]

- bit accel
- drillstring force
- friction force

Desired angular displacement trajectory for const. drill speed $\omega_r$

\[ \bar{u}(x, t) = \omega_r t - T(\omega_r)x + u_0 \]

\[ \bar{U} = -T(\omega_r) \]
Rock-on-bit friction (slope > 0 at higher speed)
Linearized model

**PDE-ODE cascade (unstable)**

\[
\begin{align*}
    u_{tt}(x) &= u_{xx}(x) \\
    u_x(1) &= U(t) \\
    u_{tt}(0) &= abu_t(0) + au_x(0)
\end{align*}
\]

where

\[
b = \frac{\partial T}{\partial \omega}(\omega_r) > 0 \quad \text{(for large RPM)}
\]
Reformulate:

drillstring torsional gradient (\textbf{twist}) + drill bit speed

\begin{itemize}
\item \textbf{twist}: \( v(x, t) = u_x(x, t) \)
\item drill bit speed: \( y(t) = u_t(0, t) \)
\end{itemize}

New set of equations (still a PDE-ODE "cascade")

\[ 
\begin{align*}
\begin{cases}
  u_{tt}(x) &= u_{xx}(x) \\
  u_x(1) &= U(t) \quad \text{(Neumann)} \\
  u_{tt}(0) &= abu_t(0) + au_x(0)
\end{cases} 
\rightarrow 
\begin{cases}
  v_{tt}(x) &= v_{xx}(x) \\
  v(1) &= U(t) \quad \text{(Dirichlet)} \\
  v_x(0) &= av(0) + aby(t) \\
  \dot{y}(t) &= aby(t) + av(0)
\end{cases}
\end{align*}
\]
• stabilize bit
• free drillstring end from bit
• dampen the freed drillstring end

Target sys.

\[ w_{tt}(x) = w_{xx}(x) \]
\[ w(1) = 0 \]
\[ w_x(0) = cw_t(0) \text{ damper (torsional)} \]
\[ \dot{y}(t) = -\delta y(t) + aw(0) \text{ damper (kinetic friction)} \]
Backstepping controller design (cont’d)

Transformation
\[ w(x, t) = v(x, t) - \int_0^x k(x, \xi) v(\xi, t) d\xi - \int_0^x s(x, \xi) v_t(\xi, t) d\xi - \gamma(x) y(t) \]

Kernel ODE coupled w/ 2 Goursat PDEs on domain \( \{0 \leq \xi \leq x \leq 1\} \)

\[
\begin{align*}
  k_{xx}(x, \xi) &= k_{\xi\xi}(x, \xi) \\
  \frac{d}{dx} k(x, x) &= 0 \\
  k_\xi(x, 0) &= ak(x, 0) \\
  &\quad + a^2 b [s(x, 0) - \gamma(x)] \\
  k(0, 0) &= a - c(ab + \delta) \\
  s_{xx}(x, \xi) &= s_{\xi\xi}(x, \xi) \\
  \frac{d}{dx} s(x, x) &= 0 \\
  s_\xi(x, 0) &= as(x, 0) - a\gamma(x) \\
  s(0, 0) &= -c \\
  \gamma''(x) &= abk(x, 0) + a^2 b^2 [s(x, 0) - \gamma(x)] \\
  \gamma(0) &= -(ab + \delta)/a \\
  \gamma'(0) &= -(ab + \delta)bc
\end{align*}
\]
Explicit expressions for the kernels

\[
\begin{pmatrix}
\kappa(x, y) \\
\sigma(x, y) \\
\gamma(x - y)
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} e^{M(x-y)} \begin{pmatrix}
a - c(ab + \delta) \\
-c \\
-(ab + \delta)/a \\
ab - cb(ab + \delta)
\end{pmatrix}
\]

\[
M = \begin{pmatrix}
-a & -a^2 b & a^2 b & 0 \\
0 & -a & a & 0 \\
0 & 0 & 0 & 1 \\
ab & a^2 b^2 & -a^2 b^2 & 0
\end{pmatrix}
\]
Feedback law (in original variables)—Fancy PD controller

\[ U(t) = [a - c(ab + \delta)] u(1, t) - k(1, 0) u(0, t) - \int_0^1 k_\xi(1, \xi) u(\xi, t) d\xi \]

\[ + c u_t(1, t) + [s(1, 0) - \gamma(1)] u_t(0, t) + \int_0^1 s_\xi(1, \xi) u_t(\xi, t) d\xi \]
Simulations (control ON at 15 sec)

Bottom velocity
Top velocity

Also available with observer measuring twist at top \( u(t, z) \).

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SLUGGING Flows in Offshore Oil PRODUCTION
Two-phase (gas+liquid) flow regimes

- **Stratified**
  - Steady flow

- **Annular**

- **Bubbly**
  - Periodic oscillations
  - Reduced production

- **Slug**
  - Periodic oscillations
  - Reduced production
Boundary control problem (with two sensing options)
Hopf bifurcation in production (unstable pneumatic spring)

- Equilibrium production
- Average production
- Min and max of the oscillations

Outlet valve opening [%] vs. Oil mass flow rate [kg/s]

- STABLE
- UNSTABLE

Desired level of production
Slugging level of production

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Drift-flux modelling

Conservation of Mass & Momentum

\[
\begin{align*}
\frac{\partial \alpha_G \rho_G}{\partial t} + \frac{\partial \alpha_G \rho_G v_G}{\partial z} &= 0 & \text{Mass of gas} \\
\frac{\partial \alpha_L \rho_L}{\partial t} + \frac{\partial \alpha_L \rho_L v_L}{\partial z} &= 0 & \text{Mass of liquid} \\
\frac{\partial \alpha_G \rho_G v_G}{\partial t} + \frac{\partial P}{\partial z} + \alpha_G \rho_G v_G^2 + \alpha_L \rho_L v_L^2 &= -\rho_m g \sin \theta(z) & \text{Combined momentum equation}
\end{align*}
\]

Algebraic equations

- Closure relations: ideal gas law, slip relation,…
- Boundary conditions: Constant gas inflow, valve equation,…
Convert to Riemann variables

3 quasilinear transport eqns:

\[ w = (u_1, u_2, v) = (\text{gas mass fraction}, \text{pressure}, \text{gas velocity}) \]

\[
\frac{\partial w}{\partial t} + A(w) \frac{\partial w}{\partial z} = S(w)
\]

\( \forall w, A(w) \) has 3 distinct eigenvalues

Physical interpretation

- \( u_1 \): pure transport [Riemann invariant] (ca. meters per second)
- \( u_2 \) and \( v \): acoustic waves (both convection speeds \( \approx 300 \text{ m/s} \))
Linearization around an equilibrium profile $\bar{w}(x)$

**PDE**

\[
\begin{pmatrix}
  u_1 \\
  u_2 \\
  v
\end{pmatrix}_t + \begin{pmatrix}
  \lambda_1(x) & 0 & 0 \\
  0 & \lambda_2(x) & 0 \\
  0 & 0 & -\mu(x)
\end{pmatrix}
\begin{pmatrix}
  u_1 \\
  u_2 \\
  v
\end{pmatrix}_x + \begin{pmatrix}
  0 & 0 & 0 \\
  \sigma_{2,1}(x) & 0 & \sigma_{2,3}(x) \\
  \sigma_{3,1}(x) & \sigma_{3,2}(x) & 0
\end{pmatrix}
\begin{pmatrix}
  u_1 \\
  u_2 \\
  v
\end{pmatrix} = 0
\]

\[\Lambda(x)\]

exp. stable

**Boundary conditions**

\[
\begin{pmatrix}
  u_1(0, t) \\
  u_2(0, t)
\end{pmatrix} = \begin{pmatrix}
  q_1 \\
  q_2
\end{pmatrix} v(0, t) \quad \quad v(L, t) = U(t)
\]
System structure and stabilization strategy

\[ u_1(t,x) \]

\[ u_2(t,x) \]

\[ v(t,x) \]

\[ U(t) \]

\[ q_1 \]

\[ q_2 \]

\[ \sigma_{21}(x) \]

\[ \sigma_{31}(x) \]

\[ \sigma_{23}(x) \]

\[ \sigma_{32}(x) \]

Transport directions
Source terms
Boundary conditions

Original system

Target system

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27 / 32
Backstepping design

Target system: \[ \gamma_t + \Lambda(x)\gamma_x + \int_0^x C(x, \xi)\gamma(t, \xi)\,d\xi = 0 \]

\[ C(x, \xi) = \begin{pmatrix} 0 & 0 & 0 \\ \sigma_{2,1}(\xi)\delta(x - \xi) + c(x, \xi) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

Backstepping transformation: \[ \gamma(t, x) = w(t, x) - \int_0^x K(x, \xi)w(t, \xi)\,d\xi \]

\[ K(x, \xi) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k^{2,2}(x, \xi) & k^{2,3}(x, \xi) \\ k^{3,1}(x, \xi) & k^{3,2}(x, \xi) & k^{3,3}(x, \xi) \end{pmatrix} \]

Control law (actuate gas flow via choke): \[ U(t) = \int_0^1 k^{31}(1, \xi)u_1(t, \xi) + k^{32}(1, \xi)u_2(t, \xi) + k^{33}(1, \xi)v(t, \xi)\,d\xi \]
Observer-based control (ON at $t = 20$ s): shown: gas pressure $u_2(t, x)$
actuation: flow rate at top $\nu(t, L)$; measurement: flow rate at bottom $\nu(t, 0)$
Mud-assisted drilling (helps take cuttings away)
Multiphase flow during drilling

- **Mud circulates** from mud pit, down through drill string, up through casing, and back into mud pit, controlled by choke valve

- Goal: regulate mud pressure to

  \[ \text{“collapse”} < \text{mud pressure} < \text{“pore”} < \text{“fracture”} \]

  to help **gas** get ingested into casing (increases penetration rate)

- **Model (physical):** separate momentum and mass conservation laws for gas and mud/rock

- **Model (in Riemann variables):** 2 positive characteristic speeds for mass transport $+$ 2 characteristic speeds for pressure waves (one positive, one negative)

- $(3+1) \times (3+1)$ hyperbolic system
Extension to \((n + 1) \times (n + 1)\) systems