Feedback Control of Bipedal Locomotion

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Videos and Papers

• [http://www.youtube.com/user/DynamicLegLocomotion](http://www.youtube.com/user/DynamicLegLocomotion)

• [http://web.eecs.umich.edu/~grizzle/papers/robotics.html](http://web.eecs.umich.edu/~grizzle/papers/robotics.html)

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- Ioannis Poulakakis
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  ROBEA Project
  France

- 2011 RAS Pioneer Award to Mark Spong

  To recognize individuals who, by virtue of initiating new areas of research, [...] have had a significant impact on development of the robotics and/or automation fields.

  • Joint elasticity in robot manipulators
  • Bilateral teleoperation
  • Normal forms for underactuated mechanical systems
  • Bipedal locomotion (Passivity and energy shaping, controlled symmetries)
Outline

- Springs in Robots
  - Mark’s problem and approach
  - Jessy’s problem
    - approach 1
    - approach 2
    - application to MABEL

- Introduction of the ATRIAS Series of Robots and MARLO (time permitting)

Manipulators: Designed to be Rigid

Harmonic drive for low backlash & high gear ratios

Manipulators with Flexible Joints

- Poor tracking?
- Vibrations in closed loop?

Stiff Spring
Joint or link side
Motor side

Harmonic drive

Fig. 1. Single-link manipulator with joint flexibility. (Courtesy M. Spong)
Manipulators with Flexible Joints

\[ D(q_1) \ddot{q}_1 + C(q_1, \dot{q}_1) \dot{q}_1 + G(q_1) + K(q_1 - q_0) = 0 \]

\[ J_1 \ddot{\theta}_1 - K(q_1 - q_0) = u \]

\[ K = \frac{1}{\epsilon} k_{\text{eff}} \]

\[ z = \frac{1}{\epsilon} (q_1 - q_0) \]

Composite Control

\[ u = u_s(q_1, \dot{q}_1) + u_c(z, \dot{z}) \]

"Rigid Model"  "Corrective Term"

Manipulators with Flexible Joints

\[ \ddot{q}_1 = -A(q_1)z - H(q_1, \dot{q}_1) \]

\[ \ddot{z} = - (A(q_1)z + B)z - H(q_1, \dot{q}_1) - Bu_c \]

Rigid Link !!!

Composite Control

\[ u = u_s(q_1, \dot{q}_1) + u_c(z, \dot{z}) \]

"Rigid Model"  "Corrective Term"
Key Papers

• Spong, Khorasani, Kokotovic, “A Slow Manifold Approach to Feedback Control of Nonlinear Flexible Systems,” ACC, 1985


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Bipedal Robots and Springs

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Natural Progression

Grizzle, Abba & Plestan 1999

Plestan, Grizzle, Abba & Westervelt 2000

Westervelt, Grizzle & Koditschek 2001

N DoF

(N-1) Actuators

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Geometric Control

\[
\dot{x} = f(x) + g(x)u
\]

\[
x^T = \Delta(x)
\]
Render surface sufficiently attractive to overcome impulse "disturbance" $\Delta$.  

HYBRID ZERO DYNAMICS
Natural Progression

Grizzle, Abba & Plestan 1999

Westervelt, Grizzle & Koditschek 2001

Rabbit

Experiments 2002-2004

Natural Progression

• Successful walking experiments
  – Model → controller design → simulation → experiment, with minimal trial and error.
  – Various speeds
  – Gait transitions (continuous & discrete)
  – Robustness to unknown loads
  – Robustness to shoves

2004 with Rabbit

7 Years of hard labor later ...
7 Years of Work to Get Here

Why Did Rabbit Not Run Well?

- Actuator saturation
  - Workspace limitations
  - Lossy powertrain
  - Nominal gait at 95% of torque limits
- Motors were asked to emulate a compliant (spring-like), high-bandwidth restoring force
  - Negative work
  - Bandwidth issues at impact
  - Ground reaction forces

Teamed up with CMU to Build a Robot with Compliance

Michigan-CMU Robot

Springs, Take 1

Planning started in late 2004
Started Analyzing Models + Series Elastic Actuators

Nominal Robot Without Compliance

\[ q = \begin{bmatrix} q_a \\ \theta \end{bmatrix} \]

\[ D(q_a)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Bu, \]

Robot + SEA

\( (2N-1) \text{ DoF} \) & \( (N-1) \text{ Actuators} \)

Highly Underactuated

\[ D(q_a)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) - BK(q_m - q_a) = 0 \]

\[ J\dddot{q}_m + K(q_m - q_a) = u_c. \]

Robot Without SEA

Feedback Equivalent (Spong, 1994)

\[ \dot{\sigma} = -\frac{\partial V}{\partial \theta}(q) \]

\[ \dot{\theta} = \frac{\alpha}{d_{NN}(q_0)} + R(q_a)\ddot{q}_a \]

\[ \dddot{q}_0 = w. \]
Robot + SEA

**Feedback Equivalent** (Isidori + Spong)

\[
\begin{align*}
\dot{\sigma} &= -\frac{\partial V}{\partial \sigma}(q) \\
\dot{\theta} &= \frac{\sigma}{d_{NN}(q_\alpha)} + R(q_\alpha)q_\alpha \\
q_\alpha^{(4)} &= w
\end{align*}
\]

**Consequences**

- Vector Rel. Degree 2 $\leftrightarrow$ Vector Rel. Degree 4
- Identical Zero Dynamics
  
  (Morris + JG TAC 2009)

Learned how to create invariant manifolds in hybrid models with many degrees of underactuation (TAC 2009)

Consequences

Design periodic orbit taking advantage of springs
Consequences

\[ y = h(q_0, \theta) \]

Stability analysis for a 1 DOF hybrid model

Springs, Take 2

Ioannis Poulakakis

TAC 2009

IROS 2007, ICRA 2008

Modeling Hierarchy

Michigan-CMU Robot

CoM

Hip

Torsional Compliance

SLIP=Spring Loaded Inverted Pendulum
Modeling Hierarchy

ASLIP

Actuator in parallel with spring (to simplify)

Modeling Hierarchy

SLIP

Modeling Hierarchy

ASLIP

Modeling Hierarchy

3 DoF

2 Actuators

Modeling Hierarchy

ASLIP
Modeling Hierarchy

Key Differences with Literature

- Massive Torso
- Hip offset from CoM
- Could not be handled by Raibert, Koditschek, Beuhler, Francois, Samson ...

Formal Embedding of the SLIP into an ASLIP Model

**Theorem** [IROS'07, TAC 2009] There exist:

1. A continuous feedback controller \( \Gamma_s \) active in the stance of the ASLIP, and an invariant surface \( Z \) (embedded) in the stance state space, such that
   \[
   f_s(x_s) + g_s(x_s)u_s = \text{SLIP stance phase model}
   \]
   \( Z \) is exponentially attractive

2. A discrete feedback controller \( \Gamma_f \) active in transitions from flight to stance, such that
   \[
   \Delta(x_f, \Gamma_f(x_f)) = \text{SLIP reset map}
   \]
   \( \mathcal{S}_{\text{vec}} \cap Z \) is hybrid invariant

Moreover,
If a controller is designed to render a particular orbit of the SLIP exponentially stable the same controller will create an exponentially stable orbit in the ASLIP closed-loop system!

Controller results available for the SLIP can be directly used in the ASLIP!

Caveat: Embedding is local due to unilateral constraints
There exists a hybrid zero dynamics such that

ASLIP

Can be controlled via two approaches:
- 1 DOF HZD
- 2 DOF HZD

Comparison of the Two Control Approaches
Identical Hybrid Controller Structure

- The SLIP embedding and the 1-DOF rigid HZD controller have the same structure, same stability proofs, though different objectives.

Identical Periodic Orbit

Comparison: Steady State

- 1-DOF HZD controller
- SLIP embedding controller
- Same leg actuator force on the nominal orbit. (<0.5% difference)

Comparison: Transients

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Control</th>
<th>Stride</th>
<th>((L^<em>), (V^</em>))</th>
<th>((L^<em>), (V^</em>))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta \theta = 1^\circ)</td>
<td>1 DOF</td>
<td>4</td>
<td>(147.15)</td>
<td>(60.18)</td>
</tr>
<tr>
<td>SLIP</td>
<td>3</td>
<td>(36.16)</td>
<td>(18.18)</td>
<td></td>
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<tr>
<td>(\delta \theta = 4^\circ)</td>
<td>1 DOF</td>
<td>13</td>
<td>(493.13)</td>
<td>(125.53)</td>
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<tr>
<td>SLIP</td>
<td>4</td>
<td>(69.16)</td>
<td>(21.20)</td>
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<tr>
<td>(\delta \zeta = +0.9%)</td>
<td>1 DOF</td>
<td>12</td>
<td>(448.21)</td>
<td>(241.76)</td>
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<tr>
<td>SLIP</td>
<td>6</td>
<td>(418.16)</td>
<td>(110.40)</td>
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</tr>
<tr>
<td>(\delta \zeta = -0.4%)</td>
<td>1 DOF</td>
<td>3</td>
<td>(73.13)</td>
<td>(40.10)</td>
</tr>
<tr>
<td>SLIP</td>
<td>3</td>
<td>(53.23)</td>
<td>(29.13)</td>
<td></td>
</tr>
</tbody>
</table>

Compliant HZD: Larger Domain of Attraction, Less Work by Actuator

Koushil Sreenath, Hae-Won Park, and Jessy W. Grizzle, *Embedding Active Force Control within the Compliant Hybrid Zero Dynamics to Achieve Stable, Fast Running on MABEL*, in review.