Homework # 5
Analysis of Discrete-time Signals

Due: March 9, 2000

A. RESPONSE OF DISCRETE-TIME SYSTEMS

1. Write a MATLAB program to determine the response of the following system:

$$y(n) = 4.5x(n) + ay(n-1)$$

where a=0.5, and the input signal x(n) is the sinewave, $x(n) = 3\sin(2\pi 0.2n)$. Assume zero initial conditions, y(-1)=0. Plot y(n), for n=1, 2, …, 200.

2. Repeat (1) for a=0.9, a=1.2, a=-0.5. What happens when a=1.2? Why?

3. Write a MATLAB program to determine the response of the following system:

$$y(n) = 4.5x(n) + 2.3x(n-2) + 4x(n-4)$$

where the input signal x(n) is the sinewave, $x(n) = 3\sin(2\pi 0.2n)$.

4. Square root algorithm
   Most computers and calculators compute the square root of a positive number A using the following recursive algorithm:

$$y(n) = \frac{1}{2}\left[y(n-1) + \frac{x(n)}{y(n-1)}\right]$$

   If we use as the input x(n) to this system (algorithm) a step function of amplitude A, then y(n) will converge after several iterations to the square root of A.

   Write a MATLAB program that implements the above algorithm to compute the square root of: 16, 4, 5 and 3. How many iterations does it take to converge to the true value assuming y(-1)=0.5? Is the algorithm sensitive to the initial conditions y(-1)?

B. IMPULSE RESPONSE

1. Write a MATLAB program to compute the impulse response of the following systems:
   (a) $y(n) = 4.5x(n) + 0.8y(n-1)$ n=0,1,…,100
   Plot the impulse response h(n) using `stem(.)`. Theoretically, what is the expression for h(n)?

   (b) $y(n) = x(n) + 0.5y(n-1) - 0.5y(n-4) + x(n-3)$
   (c) $y(n) = 4.5x(n) + 2.3x(n-2) + 4x(n-4)$
   Plot the impulse response h(n), for n=0,1,…,100.

2. Of the impulse responses computed in (1), which impulse responses are infinite in duration and which are finite? Which systems are FIR and which systems are IIR?

C. CONVOLUTION

1. Write a MATLAB program that implements the convolution sum:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

for arbitrary input signal x(n) and impulse response h(n). Implement the convolution as a function of the form: `y=convol(x,h)`. The function should take as input arguments the signal vector x(n), and impulse response h(n), and should return the output in the vector y. Assume that the signals x(n), and h(n) are zero for n<0.

2. Using the convolution program developed in (1), convolve the following sequences:
   (a)

$$x_1(n) = \{1,1,1,1,1\}$$
$$x_2(n) = \{\underset{\uparrow}{1},1,1,1,1,1,1\}$$

   (b)

$$x_1(n) = 0.5^n \qquad 0 \le \text{n} \le 100$$
$$x_2(n) = 0.9^n \qquad 0 \le \text{n} \le 100$$

3. Compute the response of the system given in question A(3) using the convolution.